Evaluation study for the Simulation of Power System Transients using Dynamic Phasor Models

T. Demiray, G. Andersson and L. Busarello

***Abstract*—In this paper the dynamic phasor approach will be used for the simulation of the IEEE 39-bus benchmark test case for dynamic studies. The dynamic phasor approach provides more accurate models than the quasi-stationary ones and, at the same time, is computationally more efficient than detailed EMTP models. Results are compared with standard electromechanical and electromagnetical models.**

***Index Terms*—Simulation, Power System Transients, Dynamic phasor approach, unbalanced conditions**

I. INTRODUCTION

For the simulation of mixed electromagnetic and electromechanical transients in power systems, various system variable representations are used. The electromagnetic transients programs often use the instantaneous value representation of system variables and system equations in the original three phases [1]. Simulations in the original three phases are accurate but inefficient due to the presence of AC quantities even at steady state conditions

Some production grade programs (e.g. SIMPOW [2]), prefer to use the DQ0 reference frame models for detailed time domain simulations. The advantage of using DQ0 based models for simulation is, that under balanced conditions (where we have only positive sequence quantities in the system) and with frequencies near to the system frequency the variations of the DQ0 transformed quantities are much slower than in the original ABC quantities, so that larger numerical integration step sizes can be used during numerical simulation. But if there are unbalanced conditions or other harmonics in the system, this advantage disappears, as the single reference DQ0 transformation is unable to simulate these harmonics efficiently.

In power systems the original phase quantities are periodical or nearly periodical. The idea behind dynamic phasors approach is now to approximate a system with nearly periodic quantities, with an appropriate set of time varying Fourier coefficients, which have slower variations but nevertheless reflect the system behavior very accurately. There has been many applications of this approach in recent years. In [3], the dynamic phasors approach has been applied to synchronous and induction machines. Reference [4] treats the application of the same approach to simulate asymmetrical faults in power systems. The application of dynamic phasors approach to power electronic based equipment has been treated in detail in [5], where a fundamental phasor TCSC model has been derived by selecting first, third and fifth Fourier coefficients as an appropriate approximation for the capacitor voltage. In [6], a systematic comparison between mostly used modeling techniques (ABC and DQ0) and the phasor dynamics approach has been made.

In these papers, since the dynamic phasors approach was mainly used to model these components and to study their behavior, small sized test systems, e.g. Single Machine Infinite-Bus (SMIB), have been used. However, efforts have not been made to implement a new type of power system simulator which based on the dynamic phasor representation of the whole power system and also simulate large power systems with the dynamic phasors approach. In [6], a MATLAB based version of this simulator was reported. The aim of this paper is to simulate the IEEE 39-bus test system for dynamic studies with the dynamic phasor models and compare their accuracy and computational performance.

The paper will be organized as follows. First the outlines of the dynamic phasors approach will be given. This will be followed by a short description of developed simulator based on the dynamic phasor representation of the power system with the model descriptions of some major components. Finally simulations will be performed on the IEEE 39-bus test system with the

- detailed EMT-models capturing electromagnetic and electromechanical transients
- detailed dynamic phasor models capturing electromagnetic and electromechanical transients
- reduced order dynamic phasor models capturing only electromechanical transients

and their accuracy and computational performance will be compared with each other.

II. OUTLINES OF THE DYNAMIC PHASOR APPROACH

The main idea of *Dynamic Phasor Approach* is to approximate a possibly complex time domain waveform $x(\tau)$ in the interval $\tau \in (t-T, t]$ with a Fourier series representation of the form

$$x(\tau) \approx Re\left\{\sum_{k\in K} X_k(t)\, e^{j\,k\,\omega\,\tau}\right\} \tag{1}$$

$$X_k(t) = \frac{1}{T}\int\limits_{t-T}^{t} x(\tau)\, e^{-j\,k\,\omega\,\tau} d\tau = \langle x\rangle_k(t) \tag{2}$$

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where $\omega = 2\pi/T$ and $X_k(t)$ is the k^{th} time varying Fourier coefficient in complex form, also called *dynamic phasor*, and K is the set of selected Fourier coefficients which provide a good approximation of the original waveform (e.g. $K = \{0, 1, 2\}$).

The dynamic phasors approach offers a number of advantages over conventional methods.

- The selection of K gives a wider bandwidth in the frequency domain than traditional slow quasi-stationary models used in Transient Stability Programs.
- The selection and variation of K gives also the possibility of showing couplings between various quantities and addressing particular problems at different frequencies.
- As the variations of dynamic phasors X_k are much slower than the instantaneous quantities x, they can be used to compute the fast electromagnetic transients with larger step sizes, so that it makes simulation potentially faster than conventional time domain EMTP-like simulation.
- At steady state the dynamic phasors X_k become constant.
- The time domain simulations of such large systems with periodically switched power electronic based components have not only a fairly high computational burden, but also give little insight into the system sensitivities used to design controllers or protection schemes. The dynamic phasors approach also allows an analytical insight into such problems, as it approximates a periodically switched system with a continuous system.

Some important properties of dynamic phasors are:

- The relation between the derivatives of $x(\tau)$ and the derivatives of $X_k(t)$, which is given in (3), where the time argument t has been omitted for clarity. This can easily be verified by differentiating the formula given in (1)

$$\left\langle \frac{dx}{dt} \right\rangle_k = \frac{dX_k}{dt} - j\,k\,\omega\,X_k \tag{3}$$

- The product of two time-domain variables equals a discrete time convolution of the two dynamic phasor sets of variables, which is given in (4).

$$\langle x\,y \rangle_k = \sum_{l=-\infty}^{\infty} (X_{k-l}\,Y_l) \tag{4}$$

III. SIMULATION FRAMEWORK AND MODELS

The simulation framework described in [6] uses the **D**ifferential **S**witched-**A**lgebraic State-**R**eset (DSAR) equations [7] for the description of the power system components‘ model behavior.

$$\begin{aligned} \frac{dx}{dt} &= f(x, y) \\ 0 &= g(x, y) \end{aligned} \tag{5}$$

Using the appropriate approximations for dynamic states x and algebraic states y in (1) and the properties (3-4), we can transform the set of f and g equations of the model into a new set of equations and get the definition of the dynamic phasor model in a new set of functions F and G as

$$\begin{aligned} \frac{dX_k}{dt} &= F_k(X_k, Y_k) - j\,k\,\omega\,X_k \\ 0 &= G_k(X_k, Y_k) \end{aligned} \tag{6}$$

where the dynamic phasors X_k become the new continuous dynamic states and Y_k the new algebraic states.

The simulation tool has been firstly implemented in MATLAB. The basic steps have been taken to implement the developed models and algorithms also in the commercial simulation program NEPLAN [8] for power system analysis. In NEPLAN, the dynamic phasor models of major power system components have been implemented. For the components with rotating masses such as synchronous machines, the dynamic phasor models are derived based on the DQ0 representation of the model equations. For other components such as transmission lines, transformer models the ABC representation of the model equations are used in the dynamic phasor model derivation.

As stated before, the key point in the derivation of the dynamic phasor models is the appropriate selection of a set X_k in (1) for an adequate approximation of the model behavior. The appropriate set of dynamic phasors depends on

- the selected reference frame (ABC or DQ0)
- the operating conditions (Balanced or Unbalanced)

If *unbalanced* conditions are of concern, the system will contain not only the positive sequence quantities but also negative sequence quantities. In the DQ0 reference frame, the fundamental frequency ac quantities in positive and negative sequence are respectively mapped as dc and second harmonic values. Due to this fact, an appropriate selection for K in (1) in the DQ0 reference frame would be $K = \{0, 2\}$ for the model derivation.

- $k = 0$ includes positive sequence quantities
- $k = 2$ includes negative sequence quantities.

The zero sequence quantities are omitted as no neutral currents are possible due to winding connections.

In the ABC reference frame, the fundamental frequency ac quantities in positive and negative sequence remain unchanged as fundamental frequency values.

IV. COMPARATIVE ASSESSMENT OF MODELS

In the case studies, three different models are compared during unbalanced faults, where two of them are based on the dynamic phasors approach.

- Detailed EMT Models - (EMT)
- Detailed Dynamic Phasor Models - (DYNPH-EMT)
- Reduced Order Dynamic Phasor Models (DYNPH-RMS)

A. Detailed EMT Models

Detailed EMT Models are used in Electromagnetic Transients Programs and capture both fast dynamics due to the electromagnetic transients and slow dynamics due to electromechanical transients. In these models, system quantities are represented by their instantaneous values $x(t)$.

B. Detailed Dynamic Phasor Models (DYNPH-EMT)

Detailed Dynamic Phasor Models capture both electromagnetic and electromechanical transients, similarly to the detailed EMT-models. This time however, system quantities are represented by their time-varying Fourier coefficients - dynamic phasors $\langle x(t) \rangle_k$.

C. Reduced Order Dynamic Phasor Models (DYNPH-RMS)

Reduced Order Dynamic Phasor Models capture only the electromechanical transients. They are equivalent to the fundamental frequency models used in the transient stability programs. In these models, e.g. stator flux electromagnetic transients of the synchronous machines and network side electromagnetic transients are neglected in the model equations.

V. SIMULATION RESULTS

The accuracy and efficiency of the detailed EMT models, detailed and reduced order dynamic phasor models under unbalanced conditions are compared in the case of the IEEE-39 Bus system. The single line diagram of the simulated power system is depicted in Figure 1. The system consists of 39 busses, 34 transmission lines, 12 two winding transformers, 19 loads and 10 synchronous machines equipped with IEEE-DC1A automatic voltage regulators. Table I shows the total number of the dynamic and algebraic states of the simulated network with the different modeling techniques. The simulated scenario is as follows. A single phase to ground fault is applied at bus N16 at 1.0 second and is cleared after 0.25 seconds. All simulations have been performed with NEPLAN [8].

A. Detailed EMT Models (EMT)

Figure 2 shows the evolution of the electrical torque of the 10 generators after the single phase to ground fault with detailed EMT models. As unbalanced conditions are of concern, negative sequence quantities will occur during unbalanced conditions. The permanent fast oscillations with the double system frequency (120 Hz) during the unbalanced fault are due to the negative sequence electrical torque. The slower electromechanical oscillations are observed in the overall evolution of the electrical torque, especially after the fault is cleared. The overall CPU simulation time for this case study with the detailed EMT models is 99.8 seconds.

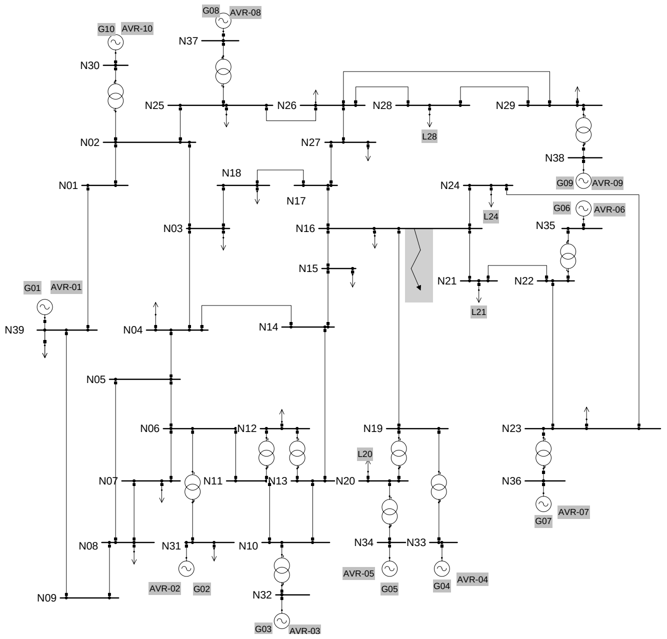

Fig. 1. Single Line Diagram of the IEEE-39 Bus Test system

	Dynamic States x	Algebraic States y
EMT	584	1909
DYNPH-EMT	1103	2560
DYNPH-RMS	245	3014

TABLE I
OVERALL NUMBER OF DYNAMIC AND ALGEBRAIC STATES OF THE SIMULATED SYSTEM

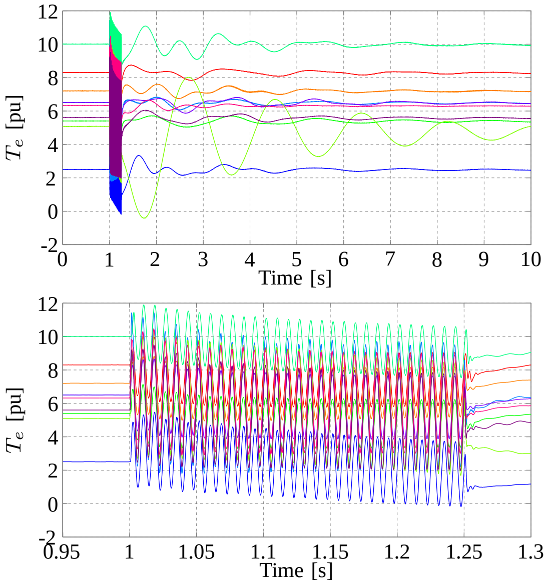

Fig. 2. Overall evolution of T_e of the 10 Generators after a single phase to ground fault at N16 simulated. The zoomed section shows the evolution during the unbalanced fault.

B. Detailed Dynamic Phasor Models (DYNPH-EMT)

Figure 3 shows the evolution of the positive sequence electrical torque $\langle T_e \rangle_0$ and Figures 4-5 show the evolution of the negative sequence electrical torque $\langle T_e \rangle_2$ of the 10 generators after the unbalanced fault. As mentioned in Section III, negative sequence quantities are mapped to the second harmonics in the DQ0 reference frame. Figures 4-5 depict, that the negative sequence electrical torque of the generators $\langle T_e \rangle_2$ arise only during the unbalanced fault and vanishes after the fault is cleared. The fast decaying oscillations with system frequency (60 Hz) in the dynamic phasor components of the electrical torque ($\langle T_e \rangle_0$ and $\langle T_e \rangle_2$) are due to the fast electromagnetic transients. The overall CPU simulation time for this case study with the detailed dynamic phasor models is 8.517 seconds.

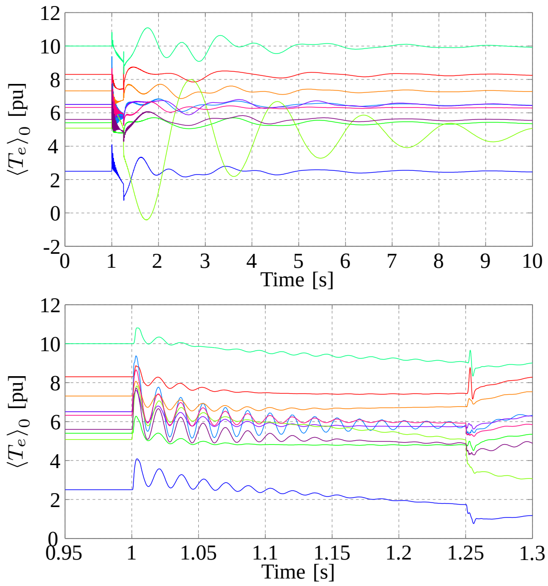

Fig. 3. Overall evolution of $\langle T_e \rangle_0$ of the 10 Generators after a single phase to ground fault at N16 simulated. The zoomed section shows the evolution during the unbalanced fault.

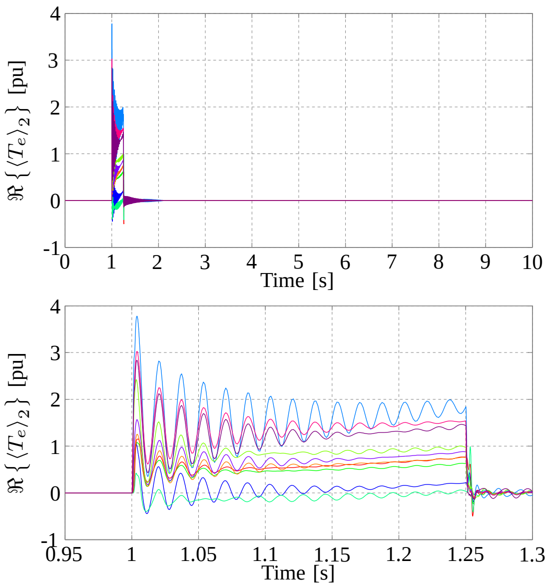

Fig. 4. Overall evolution of $\Re\{\langle T_e \rangle_2\}$ of the 10 Generators after a single phase to ground fault at N16 simulated. The zoomed section shows the evolution during the unbalanced fault.

C. Reduced Order Dynamic Phasor Models (DYNPH-RMS)

In the case of the reduced order dynamic phasor models, simulation results are shown in Figures 6-8. Also with these models the negative sequence electrical torque is captured, however only the fast oscillations due to the electromagnetic transients are not captured. The overall CPU simulation time for this case study with the reduced order dynamic phasor models is 2.523 seconds.

D. Comparative Assessment of the Results

In this section, the accuracy and efficiency of the dynamic phasor models will be compared to the detailed EMT models.

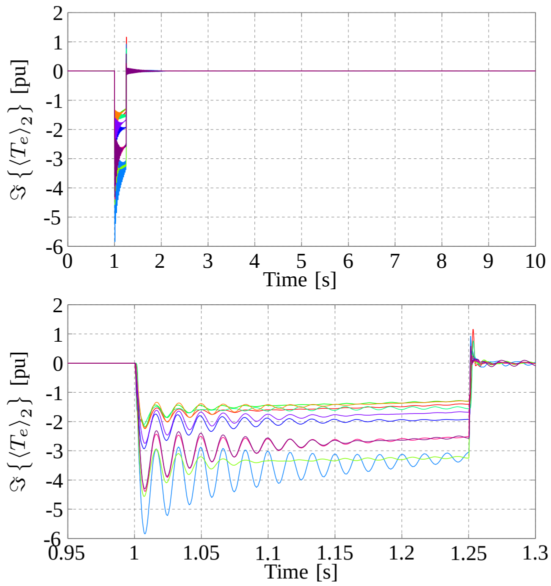

Fig. 5. Overall evolution of $\Im\{\langle T_e \rangle_2\}$ of the 10 Generators after a single phase to ground fault at N16 simulated. The zoomed section shows the evolution during the unbalanced fault.

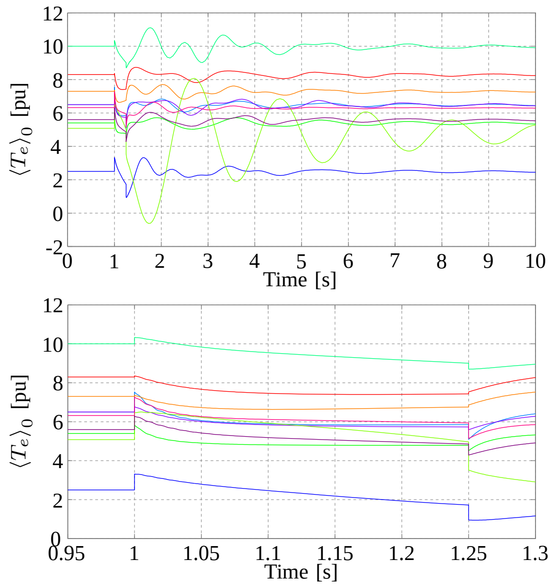

Fig. 6. Overall evolution of $\langle T_e \rangle_0$ of the 10 Generators after a three phase to ground fault at N16 simulated. The zoomed section shows the evolution during the balanced fault.

A fair comparison can be drawn only between *Detailed EMT Models* and *Detailed Dynamic Phasor Models* as they both consider electromagnetic transients. Figure 9 depicts the overall and a zoomed section of the overall simulation interval. In the case of the detailed dynamic phasor models the instantaneous value of the electrical torque T_e has been computed by using the dynamic phasor components $\langle T_e \rangle_0$ and $\langle T_e \rangle_2$ according to

$$T_e = \Re\left\{\langle T_e \rangle_0 + \langle T_e \rangle_2 \, e^{2\, j\, \omega_s\, t}\right\}$$

We see a good overall match between the results with EMT and DYNPH-EMT models, meaning that they both have nearly the same degree of accuracy. But if we compare the overall

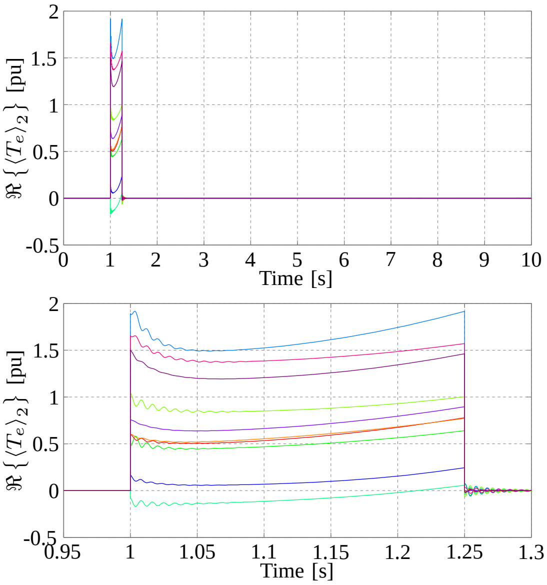

Fig. 7. Overall evolution of $\Re\left\{\langle T_e\rangle_2\right\}$ of the 10 Generators after a single phase to ground fault at N16 simulated. The zoomed section shows the evolution during the unbalanced fault.

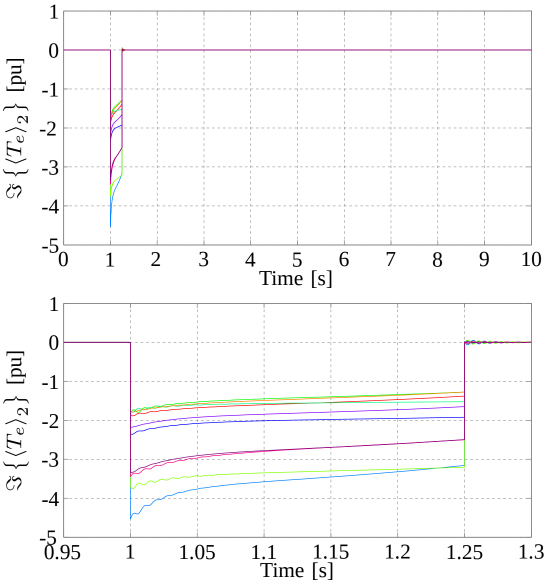

Fig. 8. Overall evolution of $\Im\left\{\langle T_e\rangle_2\right\}$ of the 10 Generators after a single phase to ground fault at N16 simulated. The zoomed section shows the evolution during the unbalanced fault.

CPU simulation times shown in Table II, simulations with the detailed dynamic phasor models are at leat 10 times faster than the detailed EMT models by keeping same degree of accuracy.

The detailed EMT models are described by the instantaneous values of the electrical quantities. In large power systems the representation of network voltages and currents with instantaneous values increases the computational burden due to the presence of AC phase quantities varying with the power frequency or system frequency (50, 60 Hz) even during steady state conditions.

Simulations with phasor dynamics are more efficient as the electrical quantities are represented by their Fourier coefficients and their variations are much slower than the instantaneous values an become dc quantities at steady state. Thus larger step sizes can be performed during simulation process.

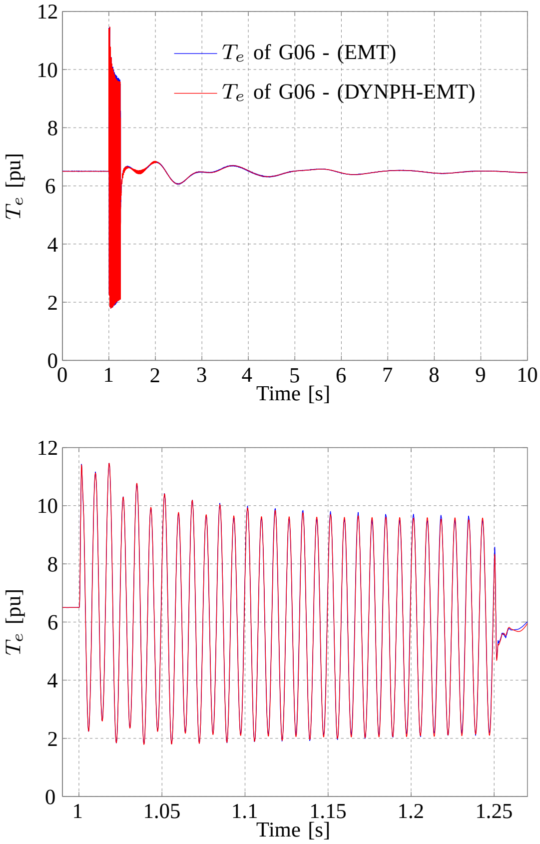

Fig. 9. Comparison of the electrical torque T_e of Generator 10 (G06) computed with EMT-models and detailed dynmaic phasor models. The zoomed section shows the evolution during the balanced fault.

As shown in Table I, even though the number of simulated variables increases with the dynamic phasors compared to the EMT models, the simulations are much faster due to the slower variations of the dynamic phasors.

The reduced order dynamic phasor models are kept out of this comparison as they neglect the fast electromagnetic transients and only capture the slower electromechanical oscillations. They are equivalent to the fundamental frequency models used in the transient stability programs. They have reduced accuracy compared to the detailed models, but are also faster in the computation time.

EMT	DYNPH-EMT	DYNPH-RMS
99.8 [s]	8.517 [s]	2.523 [s]

TABLE II
CPU SIMULATION TIMES OF THE SINGLE PHASE TO GROUND FAULT WITH DIFFERENT MODELING TECHNIQUES

VI. CONCLUSIONS

In this paper, a new type of power system simulator based on the dynamic phasor representation of the whole power system has been used to simulate a realistic power system. The simulation tool is an integral part of the commercial power system analysis program NEPLAN [8]. Results have shown that the dynamic phasor models allow an accurate and efficient simulation of the electromagnetic transients also of realistic power systems. The detailed dynamic phasor models are as accurate as the detailed EMT-models, but computationally more efficient. In the simulated IEEE-39 bus test system, the dynamic phasor models were approximately 10 times faster than the detailed EMT models.

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